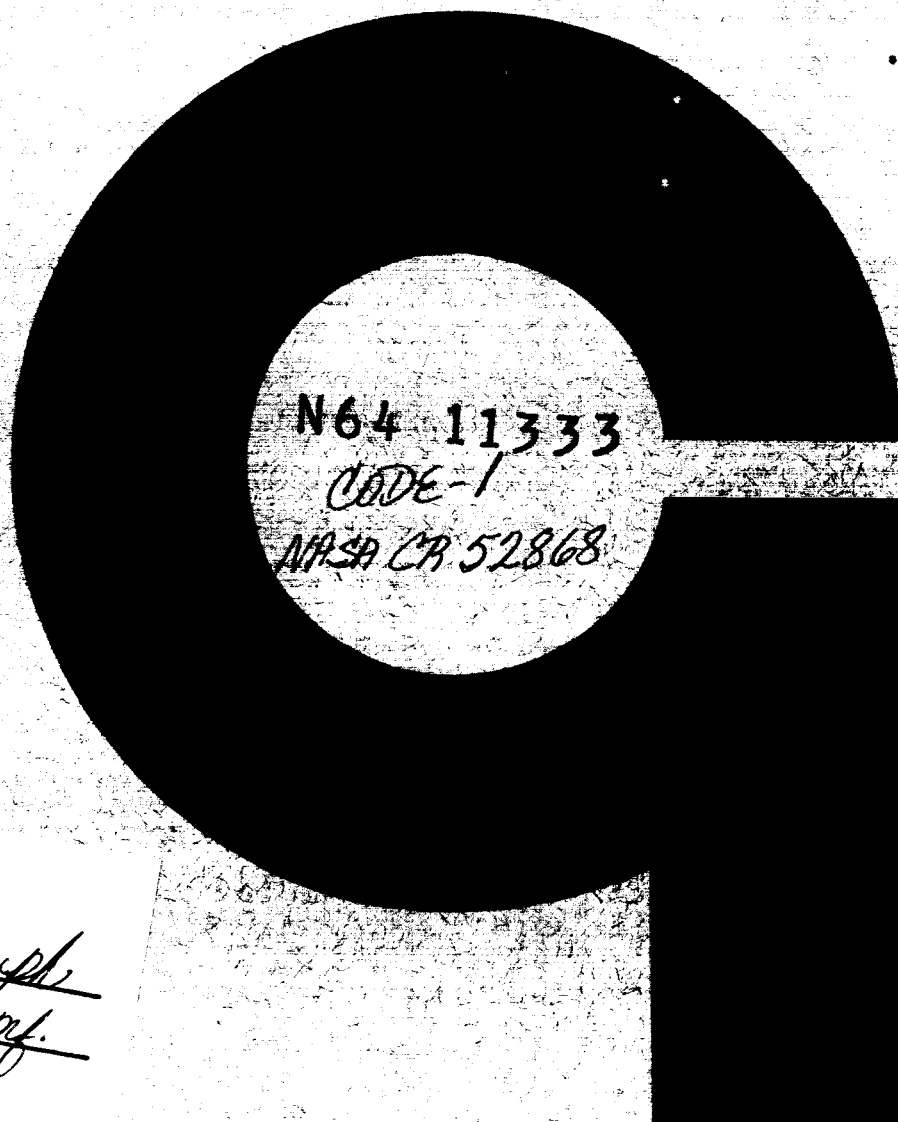


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STUDIES OF REVISED AND SUPPLEMENTARY
ATMOSPHERES AND STRUCTURES AND
VARIABILITY OF THE EARTH'S ATMOSPHERE

FINAL REPORT
CONTRACT NO. NASw-394

PREPARED FOR
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
GODDARD SPACE FLIGHT CENTER
WASHINGTON 25, D. C.

JUNE 1963

GEOPHYSICS CORPORATION OF AMERICA BEDFORD, MASSACHUSETTS

(NASA CR

GCA Technical Report No. 63-18-N

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STUDIES OF REVISED AND SUPPLEMENTARY ATMOSPHERES AND STRUCTURES AND VARIABILITY OF THE EARTH'S ATMOSPHERE

I. INTRODUCTION

This summary report presents the results of work accomplished under this NASA Contract, NAS2-394, as well as under a related Air Force Contract, AF19-(628)-1633. Because of the similarity of the effort on the two contracts, the results are being reported as a unit for continuity purposes. One major unit of work reported has in fact been jointly supported by both contracts.

This Final Report consists of abbreviated versions of GCA Technical Reports published under the above-mentioned contracts. These reports are separated into the following three groups:

- A. GCA Technical Reports published under contract NAS2-394.
- B. GCA Technical Reports published jointly under contract NASw-394 and the related contract AF19-(628)-1633, and
- C. GCA Technical Reports published under related contract AF19-(628)-1633.

the abbreviated versions of the several reports provide some of the highlights of the various efforts reported. The details including equation development discussions and analysis are available to the reader in the original reports.

Some of the problems considered during the period of the contract have been pursued only briefly and are not in condition for reporting without considerably more effort. These include a study of various scale-height concepts and their relationships to equations of other altitude-dependent atmospheric properties.

One more extensive program involving the compilation of atmospheric data and its analysis for use in supplementary atmospheres is well under way but no deductions as to systematic variability have been made. It is hoped that these efforts would be completed during an extension of the present contract.

II. SUMMARY OF REPORTS OF RECENT AND CURRENT
WORK ON MODEL ATMOSPHERES

A. SUMMARY OF TECHNICAL REPORTS PREPARED UNDER CONTRACT NASw-394

This section consists of a compilation of abbreviated versions of the following three papers prepared entirely under NASA Contract NASw-394:

- (1) A History of Standard and Model Atmospheres 1847 to 1962.
- (2) An Investigation of Some of the Constants Required in the Preparation of the Proposed Revision to the Standard Atmosphere.
- (3) Recent Contributions to the Knowledge of Atmospheric Structure and Composition Obtained from Direct Probing Experiments.

GCA Technical Report
62-6-N

**A HISTORY OF STANDARD AND MODEL ATMOSPHERES
1847 to 1962**

Raymond A. Minzner

Contract No. NASw-394

**NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
GODDARD SPACE FLIGHT CENTER
WASHINGTON, D. C.**

April 1962

**GEOPHYSICS CORPORATION OF AMERICA
Bedford, Massachusetts**

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SUMMARY OF GCA TECHNICAL REPORT 63-6-N

"A HISTORY OF STANDARD AND MODEL ATMOSPHERES 1847 TO 1962"

I. INTRODUCTION

The earth's atmosphere must have existed, essentially as we know it, for many thousands of years and yet is only within the last century that much thought was given to the nature of the atmosphere and to its variation with altitude. The development of the aneroid barometer, the balloon, the airplane, the sounding rocket, and more recently the artificial earth satellites and other space craft have each made new demands on the scientists for additional information concerning the earth's atmosphere and for tabulations of the properties of the atmosphere in the form of self consistent models. A review of the history of the development and adoption of these models of the earth's atmosphere, up to the end of 1962, is the purpose of this paper.

II. EARLY HISTORY OF EUROPEAN STANDARD ATMOSPHERES

The early history of model atmosphere is traced from isothermal models needed to calibrate aneroid barometers for measuring the altitude of land masses and manned balloons. The advantages of using more realistic temperature altitude profiles was recognized and several such non-isothermal models were introduced near the beginning of the 20th century, considerably before the development of the airplane, prior to and during World War I.

The need for greater international agreement and unity, forced upon the many nations by the revolutionary air transportation, led to considerable cooperation in matters concerning aviation, particularly in Europe.

A linearly segmented temperature-altitude profile proposed by Toussaint was generally adopted in Europe but not without some modifications by the United States largely through misunderstanding.

U. S. aeronautical standard atmospheres similar to the European standard stem from the work of Gregg and Diehl with later additions by Brombacher. These standards covered the altitude region from sea level to 20 km and were adequate until World War II when the need for an upward extension of the standard became urgent. Warfield and Grimmer contributed greatly in this effort.

During World War II the aviation interests of both Europe and the United States were drawn closely together and a unified aviation agency, The International Civil Aviation Organization, (ICAO) made its appearance with the U. S. as one of its member nations.

Revisions of U. S. and European Standard Atmospheres in keeping with an ICAO Standard Atmosphere became necessary and the U. S. led the way. Rocket instrumentation provided data for relatively accurate models up to 150 km and more recent artificial satellites provided the means for accurate extensions of the models to 700 km. The establishment of standard atmospheres in keeping with these models has become desirable and necessary.

III. EARLY U.S. AERONAUTICAL STANDARD ATMOSPHERE

The first U.S. Standard Atmosphere prepared for aircraft performance tests only was prepared by Gregg following the lower part of the Toussaint formula of $-6.5^{\circ}/\text{km}$ temperature gradient. Apparently unaware of the details of the Toussaint document Gregg established an isothermal layer 1.5° warmer than the Toussaint model and beginning at 230.77 meters below the Toussaint breakpoint of 11 km.

IV. UNIFIED U.S. STANDARD ATMOSPHERE

While aircraft performance tests were referred to an atmosphere based on an abbreviated version of the Toussaint model, altimeters apparently were still being calibrated on the basis of an isothermal atmosphere. Gregg and Diehl were instrumental in the calling of a conference in 1924 which lead to adoption of the U.S. Standard for all aeronautical purposes.

Diehl expanded the Gregg standard and appears to have been unaware of the ICAN Standard Atmosphere generally adopted in Europe in 1924, which followed the Toussaint formula in detail and used a different sea-level value of gravity acceleration from that used by Diehl.

The scope of the Diehl standard was expanded for altimetry by Brombacher when tables of altitude as a function of pressure were added. The Gregg-Diehl standard remained the U.S. Standard for a period of 27 years.

V. HIGH ALTITUDE EXTENSION TO THE DIEHL U.S. STANDARD

World War II brought about needs for extending standard atmospheres to greater altitudes and in April 1946 a committee under the chairmanship of Harry Wexler recommended the extension of the -56°C isothermal layer from 20 km to 32 km and the further extension of the model to 120 km as implemented by Warfield.

Grimminger stimulated by rocket development considered models to altitudes as great as 5000 miles but these were never adopted in their original form as standard atmospheres. Rockets, however, provided the means for the first direct probing of the high atmosphere and the data collected resulted in the preparation of the Rocket Panel Atmosphere in 1952.

VI. THE ICAO STANDARD ATMOSPHERE

The great urge for international cooperation following World War II, which resulted in the United Nations, also made its impact on aviation and science. As a successor to ICAN, and related to the United Nations, a new, more inclusive International Civil Aviation Organization (ICAO) was formed, with the United States a participating member. In July and August of 1950, a working group representing various divisions of ICAO met in Montreal, Canada, the headquarters of that organization, and prepared a proposal which led to the ICAO Standard Atmosphere, adopted originally by 52 nations. This standard represents a compromise between the Diehl U.S. Standard and the slightly different 1924 ICAN Standard and extended only to 20 km altitude.

VII. FIRST U.S. EXTENSIONS TO THE ICAO STANDARD ATMOSPHERE - ARDC MODEL 1956

With the firm establishment of the ICAO Standard Atmosphere, it became desirable to revise the U.S. high altitude aeronautical atmosphere to be consistent with the ICAO Standard as well as with the then available atmospheric data. In November, 1953, a meeting was held in Boston at which presentations were made outlining the various needs for standard atmospheres and disclosing the data upon which any choice of atmospheric model must be made. This meeting led to the preparation of the ARDC Model Atmosphere 1956 to 500 km altitude and the subsequent U.S. Extension to the ICAO Standard Atmosphere.

The extension of detailed models to high altitudes introduced new problems including the need for a variable molecular weight in the hydrostatic equation. This problem was simplified by the introduction of a new parameter, molecular scale temperature, combining temperature and molecular weight into a single parameter. These new models have been defined in terms of this new parameter.

VIII. THE SATELLITE ERA AND THE REVISION OF THE 1956 ARDC MODEL ATMOSPHERE

Late in 1957, the first calculations of atmospheric density near 220 km altitude from the drag deceleration of satellites, Sputnik $\alpha 1$ and $\alpha 2$ were published. It was evident that these densities were higher than those of the ARDC Model 1956 by a factor of 3 or more and that the ARDC Model would not provide an accurate means of determining

satellite life time. Stern Folkart and Shilling soon prepared three interim (Smithsonian) modifications to the ARDC Model with the prime objective of partially overcoming this limitation. These models had the short comings of suggesting a discrepancy between satellite drag data and lower altitude rocket data. Additional satellite and rocket data however indicated a previously unexpected rapid change in the slope of the density-altitude curve near 150 km, and the satellite and rocket data were found to be consistent.

IX. THE ARDC MODEL ATMOSPHERE 1959

Density rather than pressure became the significant property for model atmosphere preparation because it was the former which was actually being measured. Additional rocket data suggested the need for revisions at the 100 km level as well as at greater altitudes. A review of essentially all available data suggested that a model revision begin at 53 km, the top of the mesopause isothermal layer. Temperatures should be lowered in 80 to 90 km regions and should be given greater positive gradients between 100 and 200 km. These modifications resulted in the ARDC Model Atmosphere 1959, and brought about the required rapid change in slope of the density-altitude profile, noted from IGY Satellites.

X. ATTEMPTED EXTENSION OF THE ICAO STANDARD ATMOSPHERE

The publication of the 1959 Model made the current U.S. Extension to the ICAO Standard Atmosphere obsolete above 53 km. Accordingly, a meeting of the Committee on Extension to the Standard Atmosphere (COESA) was called

in January, 1960. At this meeting, it was recommended that the ARDC Model Atmosphere, 1959 be adopted as the new U.S. Extension to 700 km and that the existing U.S. Standard be submitted to ICAO as a proposed extension to the ICAO Standard Atmosphere from 20 km to 32 km. For various reasons including details in the 20 to 30 km region this proposal was unacceptable to the members of ICAO and it became desirable for the U.S. to withdraw its proposal to ICAO as well as to reconsider the adoption of the 1959 ARDC Model as the new U.S. Standard. The necessary revision to meet this objection would, of course, require essentially a complete recalculation of the 1959 Model. It, therefore, seemed wise to re-examine the entire model prior to a resubmission to ICAO, particularly because a wealth of new IGY atmospheric data had become available since the preparation of the 1959 Model.

XI. THE CURRENT REVISION OF THE U.S. STANDARD ATMOSPHERE

The above situation resulted in the reconvening of the Working Group of COESA on January 16-17, 1961 at the MIT Faculty Club under the joint chairmanship of Sissenwine of AFCRL, Wexler of USWB, and Dubin of NASA. Geophysics Corporation of America (GCA) had been appointed as a member of the parent committee of COESA in September 1960, and the writer was, in turn, appointed as the GCA representative.

The diligent work of several subcommittees and the compromises arrived at during two subsequent general meetings of the Working Group of COESA resulted in the U.S. Standard Atmosphere 1962.

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AN INVESTIGATION OF SOME OF THE CONSTANTS
REQUIRED IN THE PREPARATION OF STANDARD ATMOSPHERES

R. A. Minzner

May 1963

Contract No. NASw-394

GEOPHYSICS CORPORATION OF AMERICA
Bedford, Massachusetts

Prepared for
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Washington 25, D.C.

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ABSTRACT

The determination of the minimum number, and the best combination of fundamental physical constants necessary to define a standard atmosphere, on the basis of the tabulation of the altitude dependency of fifteen different atmospheric properties, is discussed. A recommendation for a set of defined constants based on this study is presented. The study considers the probable numerical effects on tabulated values of atmospheric properties resulting from possible future modifications of (1) the values of physical constants, (2) the definition of the absolute temperature scale and (3) the definition of the molecular weight scale.

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SUMMARY OF GCA TECHNICAL REPORT 63-15-N

"AN INVESTIGATION OF SOME OF THE CONSTANTS REQUIRED IN THE PREPARATION OF STANDARD ATMOSPHERES"

A. ARGUMENT

Some of the basic constants required in the definition of a standard atmospheres are the values of those properties required to specify the equation of state at sea-level conditions. These properties are the sea-level values of pressure, p , density, ρ , temperature, T , and mean molecular weight, M . Only three of these four quantities may be defined independently without over defining the system. The relationship of any three of these properties to the fourth is dependent upon an intrinsic proportionality factor, a physical constant known as the universal gas constant, R^* , having a value determined experimentally, and depending upon the dimensions of the several properties involved, as indicated by Equation (1).

$$\frac{p_o}{\rho_o T_o} = \frac{R^*}{M_o} \quad (1)$$

Traditionally, pressure and temperature have been assigned standard sea-level values. With any specific pure gas, M is automatically determined. Thus, with R^* universally fixed, we find density as the only undefined quantity. In the case of an unchanging arbitrary mixture of gases as in the standard atmosphere, M is not necessarily fixed but depends upon the particular mixture selected as standard. Retaining

the traditional standard sea-level values of p_0 and T_0 , one has the choice of defining either ρ_0 or M_0 .

If the standard atmosphere is limited to an altitude region within which the composition is constant, there appears to be a slight preference for defining a sea-level value of density, ρ_0 , over a sea-level value of molecular weight, M_0 . This situation stems from the fact that while p , T and ρ vary with altitude, both R^* and M would remain constant, thereby permitting these two constants to be combined into a new, single constant R , the gas constant for the particular gas mixture, air. This constant would be defined by the sea-level values of p , T and ρ in the form

$$\frac{p_0}{\rho_0 T_0} = R \quad , \quad (2)$$

and would not be subject to any revision from changes in experimentally-determined values of R^* or changes in M_0 resulting either from changes in the molecular-weight scale or from changes in observed sea-level composition of air. The definition of an altitude-dependent function of T , including T_0 , plus the definition of the constants, p_0 and ρ_0 , would determine some of the primary altitude-dependent properties of a standard atmosphere within which the composition was everywhere constant. The additional definition of some secondary constants and the altitude-dependent function of the effective acceleration of gravity, $g(h)$, would complete the definition of this limited standard atmosphere.

Such a set of constants would, of course, be at variance with agreements by the International Meteorological Organization where the standard sea-level composition and the value of M_0 are defined.

For a standard atmosphere intended to extend to great altitudes, where M no longer remains constant at the sea-level value, it has been found convenient to combine the two variables T and M into a single, new variable $R_M \equiv (T/M)M_0$ where M_0 is the sea-level value of M . Using this concept, the equation of state relates the values of p , ρ and T_M at any altitude with the sea-level value of M and the universal gas constant; i.e.,

$$\frac{p}{\rho} = \frac{R^*}{M_0} T_M \quad . \quad (3)$$

Again defining the standard sea-level values of pressure, p_0 , and temperature, this time in the form of T_{M_0} , we find with M_0 implicitly defined in the definition of T_{M_0} , that ρ_0 is the only undefined quantity. This quantity therefore, becomes the logical dependent constant and may not be defined without over-defining the system.

Recognizing that M , appearing in the definition of T_M , becomes M_0 at sea level, the value of T_{M_0} is identically equal to T_0 . Consequently, using Equation (3) evaluated at sea level, we might define T_M by the expression

$$T_M = \frac{T}{M} \frac{R^* \rho_0 T_0}{p_0} \quad , \quad (4)$$

in which case ρ_o would be arbitrarily defined, and M_o would become the dependent constant. This situation might not be unreasonable if the variable $M(h)$ were always buried within T_M as it is for the computation of the altitude variations of p , ρ , scale height H_s , specific weight ω , sound speed c_s , and mean particle speed \bar{V} .

For some quantities, however, an independently-defined function $M(h)$, or alternately an independently-defined function $T(h)$, is required in addition to $T_M(h)$ as in the following: mole volume v , number density n , mean free path L , collision frequency ν , coefficient of viscosity μ , kinematic viscosity η , thermal conductivity k , kinetic temperature T , and molecular weight M . Of the two quantities, $M(h)$ or $T(h)$, the traditional choice for independent definition has been $M(h)$. Accepting this choice, there are two possibilities for defining $M(h)$: first, in terms of a basic definition of M_o as

$$M(h) \equiv f_3 (M_o) \quad , \quad (5)$$

in which case ρ_o is not independently defined; and second, in terms of a defined value of ρ_o and other constants as in

$$M(h) \equiv f_3 \left(\frac{R^* \rho_o T_o}{p_o} \right) \quad (6)$$

in which case M_o is not independently defined. The definition expressed by Equation (5) appears to be preferable.

It is apparent that for an atmosphere which includes altitudes where $M(h) \neq M_0$, the quantity M cannot be buried within a single, new constant defined by R^*/M and applicable at all altitudes, since this ratio now is not a constant, and $M(h)$ is seen to appear independently in the definition of various properties. If ρ_0 were taken as a defined quantity, the sea-level value of the ratio R^*/M_0 , might of course be represented by $p_0/\rho_0 T_0$; but the need for separately defining R^* independently would not be eliminated by this device since R^* would be required independently in the definition of $M(h)$ as in Equation (6). Thus, the possible advantages of losing the identity of both R^* and M in a new constant R are not available in defining an atmosphere of changing composition, as it would be in one of constant composition.

Thus, it is apparent that the simplification resulting from the combining of two independent constants into a single constant which is applicable for defining a constant composition atmosphere, is no longer applicable to a variable composition atmosphere. In the latter case, the equation of state, in effect, requires the specification of any four of the five quantities in Equation (3) including the universal gas constant R^* . Whether the dependent constant is ρ_0 or M_0 is immaterial insofar as the rigor of the problem is concerned. The virtues claimed ^(1,2) for the system of constants when ρ_0 is defined do not apply in the case of an atmosphere where composition is variable.

The fact that the percentage sea-level composition is described in great detail in conventional standard atmospheres, thereby implying a value of M_0 , indicates some preference for arbitrarily defining M_0 rather than ρ_0 . Using standard values of p_0 and T_{M_0} and considering foreseeable changes in standard values of R^* and M_0 indicates that it is extremely unlikely that any future, derived, rounded value of ρ_0 would differ from the well-known value of 1.225 kg m^{-3} .

B. RECOMMENDATION

It is recommended that the U.S. Standard Atmosphere be defined in terms of a set of constants including the following:

(1) The internationally accepted standard sea-level values of $p_0 = 101325.0 \text{ nt m}^{-2}$ (1013.250 mb) and $T_{M_0} = T_0 = 228.15^\circ\text{K}$,

(2) the best value of R^* as adopted by the National Research Council,

(3) a value of M_0 consistent with an accepted standard sea-level composition and computed in terms of a molecular weight scale for which carbon 12 = 12,

(4) a value of ρ_0 defined by $p_0 M_0 / R^* T_{M_0}$, and

(5) the best value of other needed constants as adopted by the National Research Council.

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GCA Technical Report No. 63-7-N

**RECENT CONTRIBUTIONS TO THE KNOWLEDGE OF
ATMOSPHERIC STRUCTURE AND COMPOSITION
OBTAINED FROM DIRECT PROBING EXPERIMENTS**

H. E. LaGow

R. A. Minzner

Contract No. NASw-394

**NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
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SECTION 1

INTRODUCTION

The more significant aspects of the recent contributions reviewed include the following: (1) The average atmospheric temperature and mass density in the isothermal region above 200 to 300 km are both lower than corresponding values observed during the IGY, and both properties have significant short-term temporal variations; (2) The 70 - 100 kilometer temperature profile through the mesopause has been accurately measured for the first time; (3) The base of the altitude region of atomic oxygen dominance (where atomic oxygen comprises 50% or more of the total atmosphere) has been observed to be much lower than that generally assumed immediately after the IGY; (4) Helium has been recognized as being the dominant atmospheric constituent for a limited altitude region between the region of atomic oxygen dominance and the region of hydrogen dominance; (5) Winds are being measured synoptically up to 70 km altitude; (6) A 1962 U. S. Standard Atmosphere to 700 km altitude has been issued; it reflects the new temperature and density data but was prepared too soon to utilize the new O_2 dissociation information.

SECTION 2

TEMPERATURE, PRESSURE AND DENSITY

Two significant direct-probing satellite experiments serve to corroborate independently the satellite-drag densities reported separately by Jacchia (1963). These are ionization-gauge observations (McIsaac et al. 1962), and microphone-gauge observations (Sharp et al. 1962). The latter indicated a 4:1 diurnal variation in density at 550 km for Wallops Island latitudes and was in agreement with simultaneous satellite-drag results.

A rocket-grenade, sound-velocity experiment (Stroud et al. 1960) has detected large seasonal variations in temperature at Fort Churchill up to the temperature minimum at 80 mm altitude, and unpublished results of the same experiment show winter conditions above Wallops Island to be similar to simultaneously observed Fort Churchill conditions. Temperature and the related pressure-and density-altitude profiles were observed at Fort Churchill by the pitot-static-tube method (Ainsworth et al. 1961). Ainsworth's detailed and very accurate data provided a degree of fine structure in temperature from 30 to 110 km altitude not previously available. (Nordberg and Stroud 1961) measuring November temperatures

to about 80 km over Guam found little difference from the average White Sands values. A simple and relatively inexpensive rocket-sonde system has provided synoptic observations of temperatures up to about 48 km altitude (Joint Sci. Adv. Group to Met. Rocket Net. 1961).

High-altitude temperatures have also been inferred from measurements of the following:

- (a) Electron temperatures in certain altitude regions where thermal equilibrium is believed to exist in the quiet ionosphere, (Brace et al. 1962), (Serbu et al. 1961), Spencer et al. 1962).
- (b) The altitude rate of decrease of natural log of electron number densities (Bauer 1962) above the F_2 maximum.
- (c) The temperature of molecular and atomic ions (Nagy et al. 1962); (Bauer & Bourdeau 1962); and
- (d) Spectral observations of emission lines from sodium vapor ejected into the upper atmosphere (Blamont 1962). (Blamont 1962) shows temperature above Wallops Island at altitudes from 210 to 420 km to be isothermal at $950 \pm 100^\circ\text{K}$.

The results of all the above measurements generally have been to confirm the existence of an isothermal region above about 300 km with a varying temperature which depends on time of day and upon other conditions.

SECTION 3

COMPOSITION

Rocket measurements of atmospheric composition have been performed by means of neutral-particle as well as charged-particle mass spectrometers, and ultraviolet monochromators. Composition has also been inferred from observations of electron and ion densities.

From measurement of the neutral particle ratio A/N_2 Meadows and Townsend (1960) showed diffusive separation to occur in the 100 to 120 km region.

Johnson and Holmes (1960) and Johnson (1961), and Taylor and Brinton (1961) reporting on ion composition from radio frequency mass spectrometer observations over Fort Churchill and Wallops Island respectively all report major ion constituents to be 16^+ , 30^+ , and 32^+ AMU, while ions of atomic mass 14^+ , 18^+ , and 28^+ were detected as minor constituents.

Schaefer (1961), (1962), using a massenfilter has reported values of O/O_2 ratio of 0.5 at 110 km, 1.2 at 120 km and 2 at 130 km altitude, and claims these results to be in general agreement with ultraviolet absorption results reported by Kupperian et al. (1960).

Hinteregger (1962) analyzing his extreme ultraviolet monochromator measurements reports atomic nitrogen to be a minor atmospheric constituent below 225 km altitude, and his data shows that atomic oxygen becomes the dominant atmospheric constituent above approximately 150 km altitude.

These results by Kupperian et al., Hinteregger, and Schaefer appear to confirm each other, and can be expected to markedly change the molecular weight and temperature-altitude profile of current model atmospheres.

Hanson and McKibbin (1961) have shown, from ion trap measurements of ion number density to 700 km, that above 350 km these data could represent mass 16^+ ions at a constant temperature of 1240°K . The predominance of O^+ in this altitude region was also derived from Explorer VIII data by Bourdeau et al. (1961).

Following the suggestion by Nicolet in April 1961, published in 1962, that a region of helium dominance between 750 and 1500 km could explain the Echo Satellite performance and that such an atmosphere could be demonstrated to exist on the basis of diffusive separation (Nicolet 1961), (Sauerman & Herzog 1961), various investigators began looking for helium specifically.

Ion density data to 5500 km (Hale 1961) were interpreted by Hanson (1962) as indicating the following: a predominance of O^+ ions up to 1200 km altitude, He^+ ions from 1200 to 3400 km, and H^+ ion (protons)

above 3400 km. Bourdeau et al. (1962a), (1962b), substantiated the Hanson finding by obtaining a value of $1.3 \pm .03$ for He^+/O^+ at 1630 km by direct sampling techniques. Taylor et al. (1962) measuring H^+ and He^+ ions directly with an RF mass spectrometer found significant quantities of both ions from 400 to 940 km.

Bauer (1962 & 1962) suggests that the thickness of the helium ion belt varies with temperature from 2000 km at 1600°K to about 200 km at 600°K . This has been corroborated by recent experimental data being reported by Bourdeau (1962).

Unfortunately no direct measurements of neutral He and hydrogen have yet been made.

SECTION 4

WINDS

During the past three years winds have been measured synoptically for the first time to altitudes of 70 km (Joint Sci. Adv. Grp. to the Met. Rocket Net. 1961) by means of radar tracking of chaff, silvered balloons and parachutes (Rapp 1960), (Jenkins 1962) ejected from hundreds of small inexpensive rockets launched in coordinated series from several North American launch sites (Smith 1960, 1962), (Masterson et al. 1961) and (Keegan 1962). The data is centrally collected and distributed by the Meteorological Rocket Network Committee for analysis (Batten 1961), (Appleman 1962).

At higher altitudes up to about 200 km, wind measurements are not yet routine but have greatly increased in number. The several methods employed include rocket grenades (to altitudes of about 90 km) (Stroud et al. 1960), (Nordberg and Stroud 1961), tracking of rocket exhaust trails (to 140 km) (Woodbridge 1962), (Aufm Kampe et al. 1962), (Pochtzhke H) and other especially formed vapor trails usually of alkali metals (Manring et al. 1961), and one pitot static tube experiment (Ainsworth et al. 1961).

Among the more significant developments has been the series of coordinated observations by the same method at various locations (GCA 1962) and by various methods at the same location (Nordberg and Smith 1962). Winds deduced from these data compare favorably (Manring et al. 1962). In November-December 1962 eight rocket-launch sites around the world were used for the launching of 23 high-altitude rockets with sodium-vapor and grenade payloads in a cooperative effort to establish global wind structure. Sound-grenade rockets were launched nearly simultaneously from Wallops Island, Fort Churchill, and Australia. Data from this operation is not yet available.

Although not enough data exists to construct circulation systems in the atmosphere above 70 km, the several experiments have measured other interesting though not completely understood phenomena. These include high and persistent wind shears in the region around 100 km altitude, wind velocities in excess of 100 meters per second above the shear region, and turbulence below this shear region (Blamont and Jager 1962), (Cote 1962). The circumpolar vortex characteristic of high latitude wintertime conditions has been observed to penetrate as far south as Wallops Island 38°N in February 1961 (Nordberg and Smith 1962).

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**B. SUMMARY OF REPORTS PREPARED JOINTLY UNDER CONTRACT NASw-394
AND THE RELATED AIR FORCE CONTRACT AF19(628)-1633**

TEMPERATURE DETERMINATION METHOD
FOR PLANETARY ATMOSPHERES*

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ABSTRACT

Number-density-versus-altitude data for two gases having large differences in molecular weight permit the accurate determination of the temperature of planetary atmospheres in diffusive equilibrium. A well-known equation relating temperature and number density of a single gas when applied alternately to the heavy and light gases determines a temperature-altitude profile with associated uncertainties which are minimized by an iterative process.

A new equation simultaneously involving both the light- and heavy-gas number densities permits accurate temperature calculations in a single-step process. This dual-gas equation applied to the upper half of a region of observed number-density data, together with the heavy-gas-down equation applied to the lower half of the region provides a minimum-error, temperature-altitude profile over the entire altitude range of number-density data. A rigorous error analysis demonstrates how the integration limits may be optimized. The earth's atmosphere characterized by the 1962 standard atmosphere serves as a numerical example. Simulated observational data of helium and argon number densities (stemming from a micropause

at 120 km) as would be provided by presently available mass spectrometers and conventional telemetering systems - each with specified limitations - yields temperatures with uncertainties approximately equal to the number-density uncertainty at the corresponding altitudes between 150 and 250 km altitude, and with slightly greater uncertainties to altitudes as great as 700 km.

SUMMARY OF GCA TECHNICAL REPORT 63-10-N

"TEMPERATURE DETERMINATION METHOD FOR PLANETARY ATMOSPHERES"

I. INTRODUCTION

Consideration is given to a hypothetical mass-spectrometer experiment which measures number densities of a light and heavy gas. When flown in an orbiting satellite or a vertically flown sounding rocket in any planetary atmosphere, this instrument measures number-density data, yielding kinetic temperature over the entire altitude range of significant observations as well as other results which are not considered. Although the numerical work has been performed assuming a mass spectrometer with a noise level equivalent to a number density of $1 \times 10^{10} \text{ m}^{-3} (1 \times 10^4 \text{ cm}^{-3})$, the results can be applied to a mass spectrometer with an arbitrary noise level. The discussion is confined to the properties of several forms of the temperature-altitude, number-density equations which are given in the following section.

II. THE EQUATIONS

The four equations of interest are:

Single-Gas Equations;

$$T_b = \frac{n_a^*}{n_b^*} T_a + \frac{GM^*}{R} \int_b^a \frac{n^*}{n_b^*} dh, \quad (1)$$

$$T_a = \frac{n_b}{n_a} T_b - \frac{GM}{R} \int_b^a \frac{n}{n_a} dh, \quad (2)$$

Double-Gas Equations;

$$T_a = \frac{1}{n_a/n_b - n_a^*/n_b^*} \left[\frac{GM^*}{R} \int_b^a \frac{n^*}{n_b^*} dh - \frac{GM}{R} \int_b^a \frac{n}{n_b} dh \right], \quad (3)$$

$$T_b = \frac{1}{n_b^*/n_a^* - n_b/n_a} \left[\frac{GM^*}{R} \int_b^a \frac{n^*}{n_a^*} dh - \frac{GM}{R} \int_b^a \frac{n}{n_a} dh \right], \quad (4)$$

where

n_a^* is the number density of a heavy gas at the reference level,

n_b^* is the number density of a heavy gas at the running altitude

b (below "a"),

T_b is the temperature at altitude b,

Gdh is an increment of geopotential,

h is geopotential altitude,

R is the universal gas constant, and

M^* is the molecular weight of the heavy gas.

The asterisks on n , n_a , n_b and M indicate the association with values

• for a heavy gas like argon.

Equation (1) is well known and has been used by Elterman and others to deduce temperature from total number-density data at altitudes below a reference level for which an initial temperature T_a must be assumed or determined independently. Equation (2) is an inverse form of the

first, and for total number-density data of the atmosphere (molecular weight 29), this form has long been known to be useless. For a light gas, like helium, however, this equation is, in fact, a very useful one for obtaining temperatures above a reference level "b" for which altitude a value of T_b is assumed or is available from independent observations. The various symbols have the same significance as in the first equation except that the absence of the asterisk designates a light gas like helium.

Equations (1) and (2) may be used iteratively to successively improve the computed values of temperature between altitudes a and b. These equations may also be combined to eliminate T_a as in Equation (3), or to eliminate T_b as in Equation (4). These double-gas equations provide the means for a single-step temperature determination for any point over the entire altitude interval b to a. It has been shown analytically that the iterative use of Equations (1) and (2) in the limit reduced exactly to the two double-gas equations.

It has also been shown analytically that for altitude "b" sufficiently below altitude "a", Equation (3) reduces to Equation (1). Similarly for "a" sufficiently greater than "b", Equation (4) reduces to Equation (2).

III. NUMBER DENSITY DATA

The helium and argon number-density data shown in Figure 1 have been used to analyze numerically Equations (1) through (4).

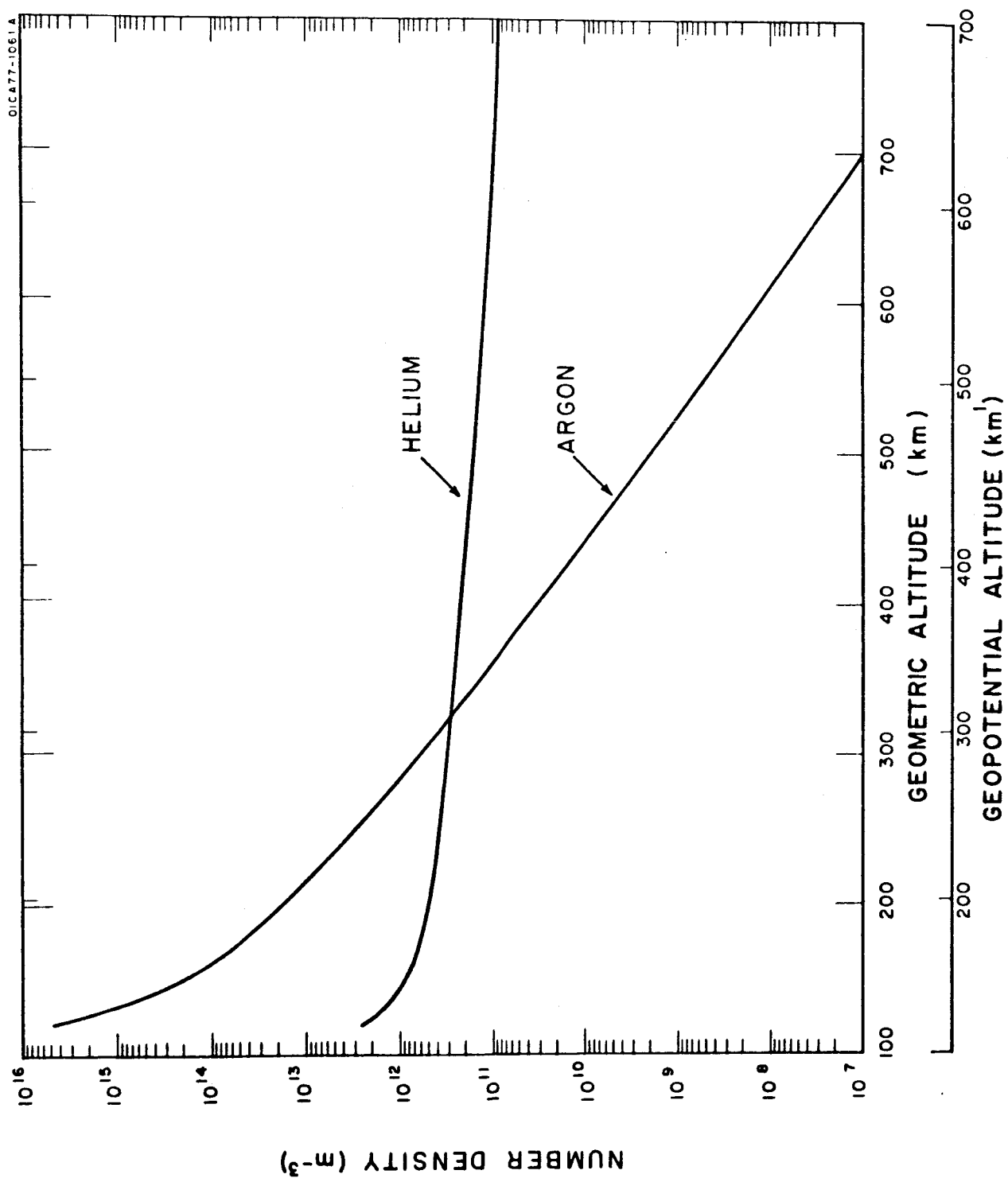


Figure 1. Number density of helium and argon vs altitude.

These values are computed from the standard-atmosphere temperature profile on the basis of a micropause level of 120 km. According to this model, argon number-density values vary more than a factor of 10^5 between 150 and 450 km geometric altitude, the altitude where argon signal-to-noise ratio falls below unity in the mass spectrometer being considered. Helium, on the other hand, varies by less than a factor of 10 between 150 and 700-800 km, an altitude where n is still considerable above the mass-spectrometer noise level.

IV. EVALUATION OF EQUATIONS

Figure 2 shows the evaluation of the single-gas down equation where the heavy line is the locus of the values of temperature and the lower dashed lines are the loci of values of the integral terms for argon and helium as labeled. Because the equations are defined in terms of geopotential altitude, the figures are also in terms of geopotential, with the units km'. The value of the ratio term $\left(\frac{n_a^*}{n_b^*}\right) T_a$ is represented by the difference between the temperature line and the argon integral line at any altitude coordinate. The value of this term drops to a negligibly small value of less than 1% of T_b at altitudes below 300 km. For helium, with reference level at 700 km, the ratio term $\left(\frac{n_a}{n_b}\right) T_a$ never becomes negligibly small within the region of interest, and is still 18% of T_b at 150 km.

If T_a is assumed to be 0°K with an argon calculation, the value of the ratio term is zero. The computed values of T_b would, therefore,

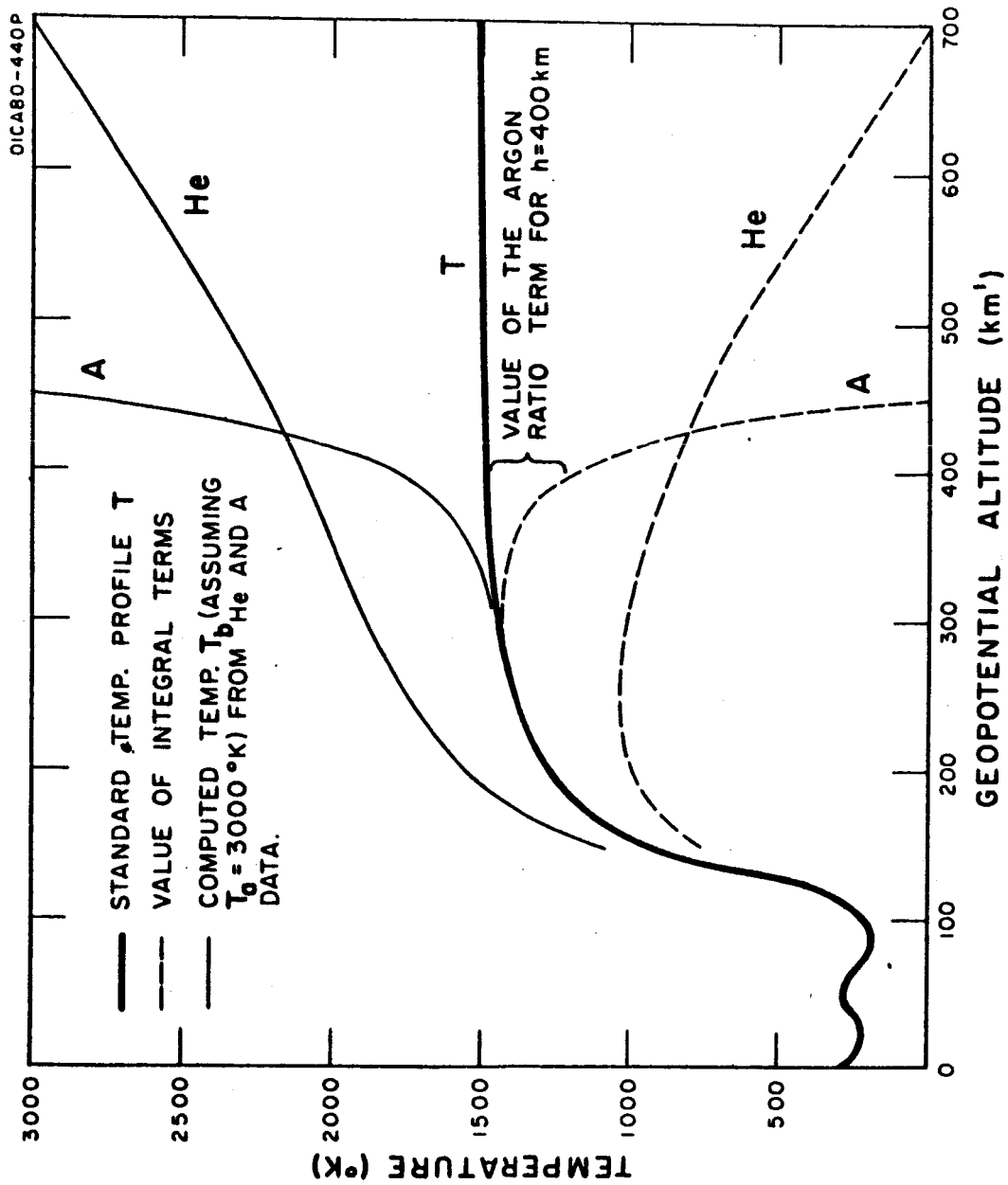


Figure 2. Contribution of the two terms of the single-gas-down temperature equation for both helium and argon number densities.

follow the dashed line labeled A and accurate values of T_b would be obtained at altitudes below 300 km'. If T_a is assumed to be 3000°K in an argon calculation, the value of the ratio term is 3000° at 450 km' and the computed values of T_b will follow the solid line labeled A. Again, accurate values of T_b will be obtained below 300 km'. For a similar range of assumed values of T_a with helium number-density data, no realistic values of T_b are forthcoming for any part of the altitude region considered. The single-gas-down equation is obviously unsuitable for a gas as light as helium.

Figure 3 presents an evaluation of the single-gas-up equation on a semi-log plot made necessary by the large range of values. The heavy line again is temperature. The dashed line, representing the value of the argon integral term, increases indefinitely with increasing altitude as does the argon ratio term represented by the light solid line labeled A. The temperature T_a is the difference between the two terms whose values are represented by these two lines. Such a relatively small difference (of the order of 10^3) between two very large values (i.e., 10^8 at 450 km') is very inaccurate. An uncertainty of 10°C in T_b at 150 km' propagates to an error of 10^6 degrees C at 450 km'. It is apparent that the single-gas-up equation is not suitable for a gas as heavy as argon.

With helium on the other hand, the ratio term and the integral terms both have values only slightly larger than the temperature. In

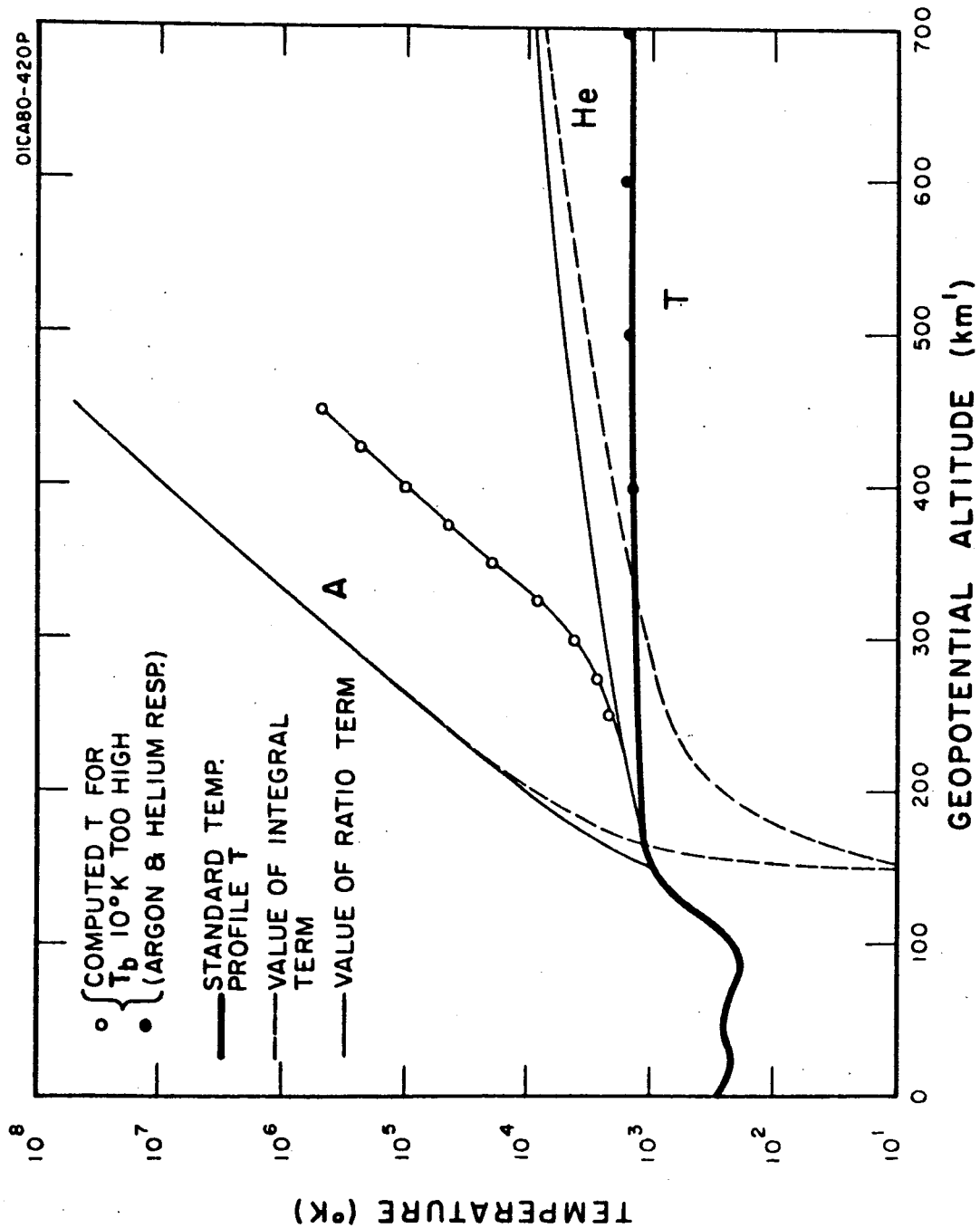


Figure 3. Contribution of the two terms of the single-gas-up temperature equation for both helium and argon number densities.

this instance an error of 10°C at $150\text{ km}'$ propagates to an error of only 100°C at $700\text{ km}'$. This equation is apparently quite suitable for a light gas calculation.

Figure 4 demonstrates the evaluation of the double-gas-up equation. Here the coordinate scales are all linear because of the more limited range of values. The heavy line is temperature and the light solid line is the dimensionless ratio factor. The dashed line for the argon integral rises rapidly and becomes constant above about $300\text{ km}'$. The helium integral term continues to rise with altitude but always stays below the argon value. With some manipulation this equation may be shown analytically to be light-gas dominated, i.e., at altitudes above $300\text{ km}'$, it becomes essentially equal to the light-gas-up equation. This situation is not readily demonstrated graphically with the form of the double-gas equation as presented.

Figure 5 shows the evaluation of the double-gas-down equation. Like an earlier figure, this one is also presented in a semi-log plot because of the large range of values. The heavy line represents temperature. The two dashed lines are the values of the argon and helium integrals as labeled. Again the light solid line represents the locus of the dimensionless ratio factor. The difference between the integral terms multiplied by the ratio factor yields T . In this form the relationship of the double-gas-down to the heavy-gas-down equation is not evident. If, however, each integral term is separately multiplied

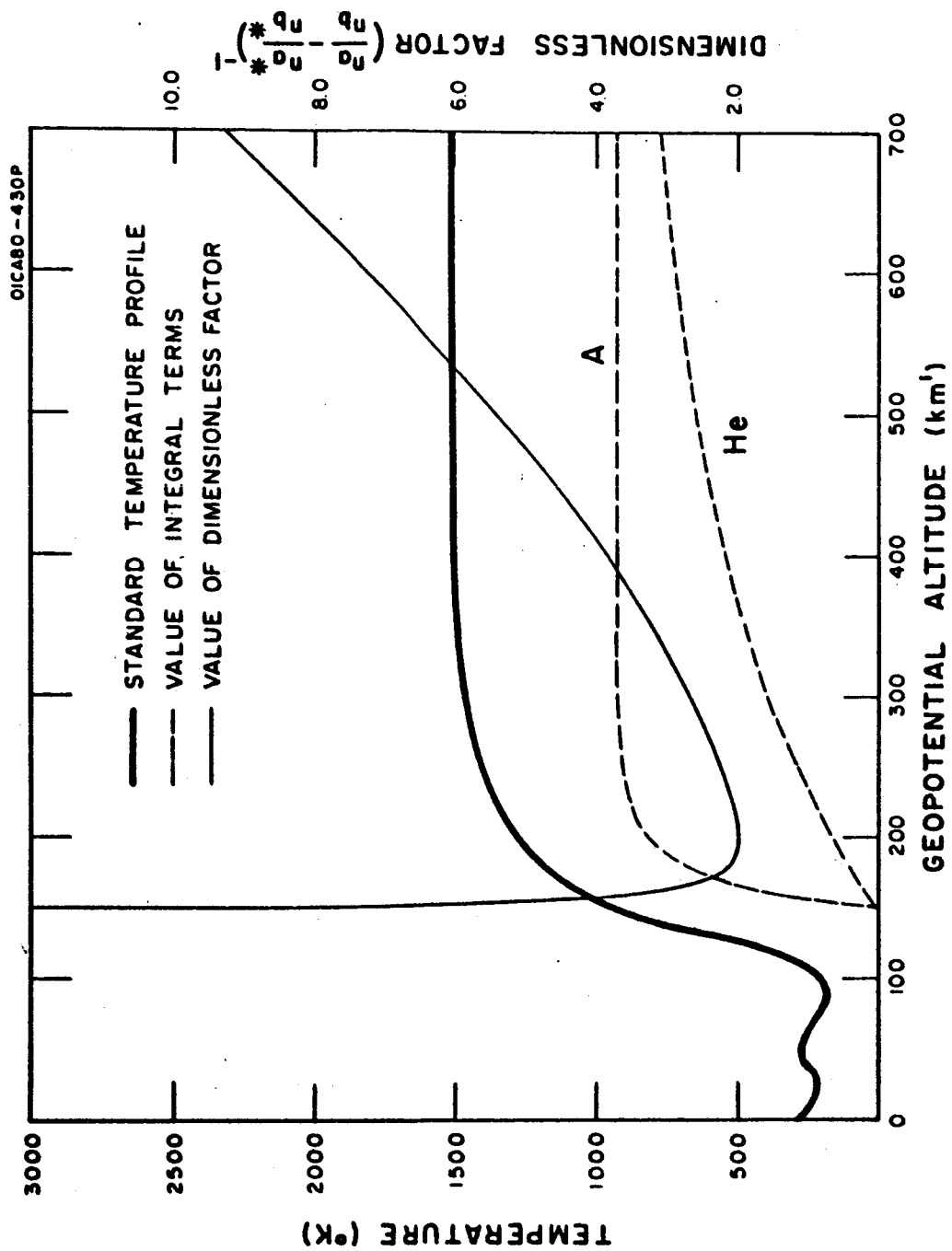


Figure 4. Contribution of the two integral terms and the ratio factor of the double-gas-up temperature equation for helium and argon number densities.

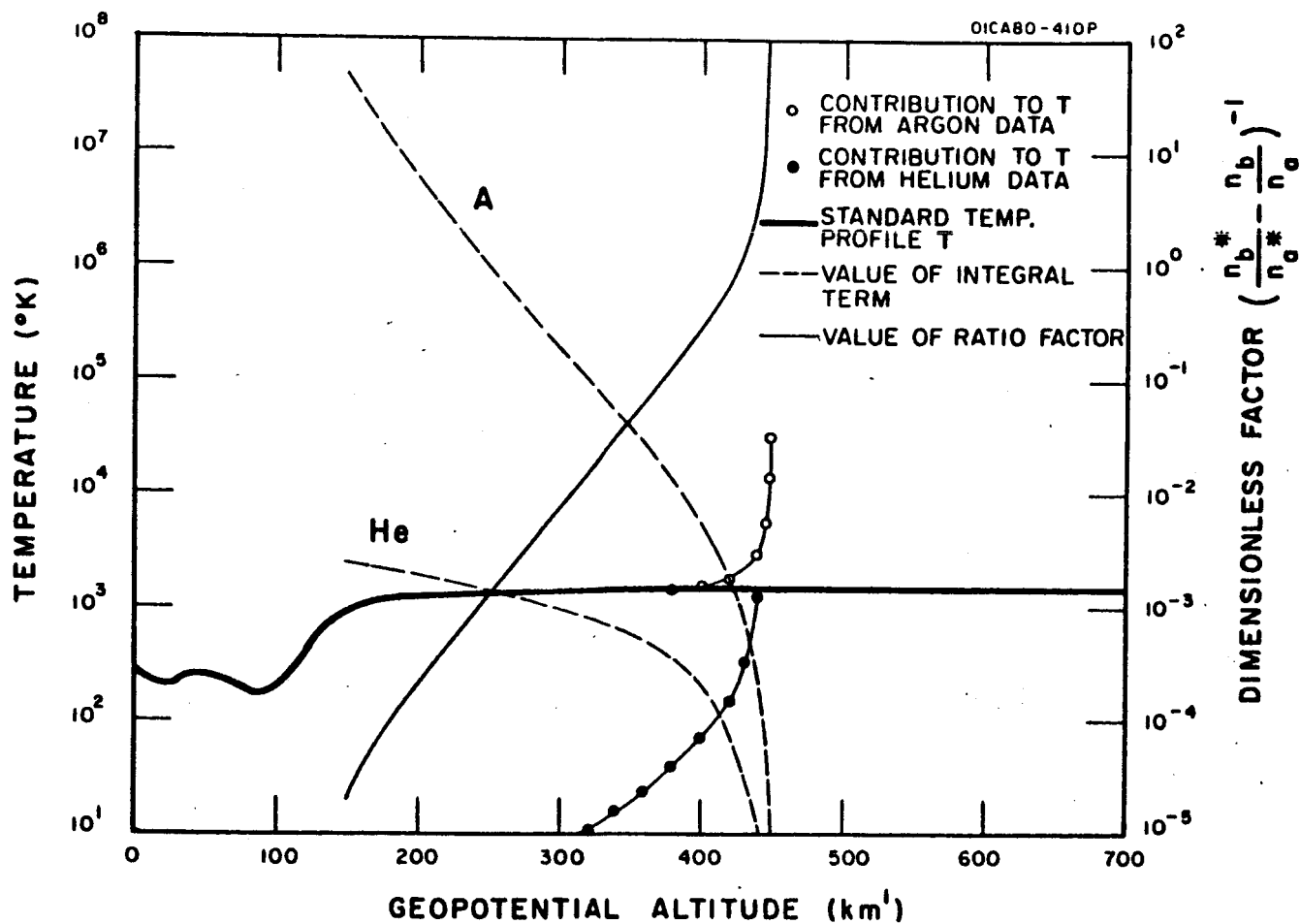


Figure 5. Contribution of the two integral terms and the ratio factor of the double-gas-down temperature equation for helium and argon number densities.

by the ratio factor the resulting values are the open circles for the argon term and the solid circles for the helium term. As b moves below 300 km' it is evident that the contribution from the helium term becomes small and the double-gas-down equation becomes argon dominates, i.e., it becomes essentially equal to the heavy-gas-down equation at low altitude, as demonstrated analytically.

V. ERROR CONSIDERATIONS

The temperatures from each of Equations (1) through (4) should, in principle, be the same at a given altitude. The uncertainties associated with each equation are very different, however, and the various equations exhibit altitude regions of optimum utility. Using the gaussian method for error determination, one obtains an expression for the uncertainty in temperature as determined by each of Equations (1) through (4) in terms of uncertainties in number densities, altitudes and reference-level temperatures. On the assumption that the uncertainty in a number-density-altitude data point may be entirely lumped into the number-density value the altitude-uncertainty portion in each of the uncertainty expressions was eliminated. The resulting uncertainty expressions follow:

$$\left(\delta T_{\underline{b}}^{\underline{a}} \right)_h = \frac{1}{n_b^*} \sqrt{u^2 + \left(n_a^* \delta T_a \right)^2} , \quad (5)$$

$$\left(\delta T_{\underline{b}}^{\underline{a}} \right)_\ell = \frac{1}{n_a} \sqrt{v^2 + \left(n_b \delta T_b \right)^2} , \quad (6)$$

$$\left(\delta T_b^a\right)_d = \frac{1}{n_a/n_b - n_a^*/n_b^*} \sqrt{\frac{u^2}{(n_b^*)^2} + \frac{v^2}{(n_b)^2}}, \quad (7)$$

$$\left(\delta T_b^a\right)_d = \frac{1}{n_b^*/n_a^* - n_b/n_a} \sqrt{\frac{u^2}{(n_a^*)^2} + \frac{v^2}{(n_a)^2}}; \quad (8)$$

where

$$u^2 = (\delta n_a^* T_a)^2 + (\delta n_a^*)^2 T_a \frac{2GM^*}{R} \Delta h_a + \left(\frac{GM^*}{R}\right)^2 \sum_{j=2}^q (\delta n_j^* \Delta h_j)^2 + (\delta n_b^* T_b)^2, \quad (9)$$

$$v^2 = (\delta n_a T_a)^2 + (\delta n_a)^2 T_a \frac{2GM}{R} \Delta h_a + \left(\frac{GM}{R}\right)^2 \sum_{j=2}^q (\delta n_j \Delta h_j)^2 + (\delta n_b T_b)^2, \quad (10)$$

and where Equations (5), (6), (7) and (8) represent the uncertainties of Equation (1), (2), (3) and (4), respectively.

The notation associated with the δT indicates the basic equation to which it refers, i.e., the superscript a and the subscript b indicate the limits of integration or summation with the underline indicating the reference level. The subscripts h , ℓ and d represent heavy gas, light gas and double gas respectively.

In order to evaluate the temperature-uncertainty equations the following number-density uncertainty functions were adopted:

$$\frac{\delta n^*}{n^*} = \frac{10^{10}}{n^*} + .01, \quad (11)$$

and

$$\frac{\delta n}{n} = \frac{10^{10}}{n} + .01. \quad (12)$$

These are based on a mass-spectrometer sensitivity[†] of 1×10^{10} particles/m³ ($1 \times 10^4/\text{cm}^3$) and an overall reading uncertainty of one percent. A graph of these number-density uncertainty functions in Figure 6 shows that the signal-to-noise-ratio considerations dominate the uncertainty except at very low altitudes.

Families of curves of temperature uncertainty versus altitude were computed for each of the four temperature-uncertainty equations using the above-mentioned number-density error functions and various values of reference altitude. It was found that for upward-type calculations the temperature uncertainty at any altitude is decreased to the point where diffusive separation no longer applies. A low-altitude limit of 150 km was arbitrarily applied. For the downward-type calculations the temperature uncertainty at any altitude decreases as the reference altitude increases up to a point where the increasing uncertainty in number density dominates the expression such that further increase in altitude produces increasing errors. For the assumed argon number-

[†]The sensitivity in this instance is the value for which the signal-to-noise-ratio becomes unity.

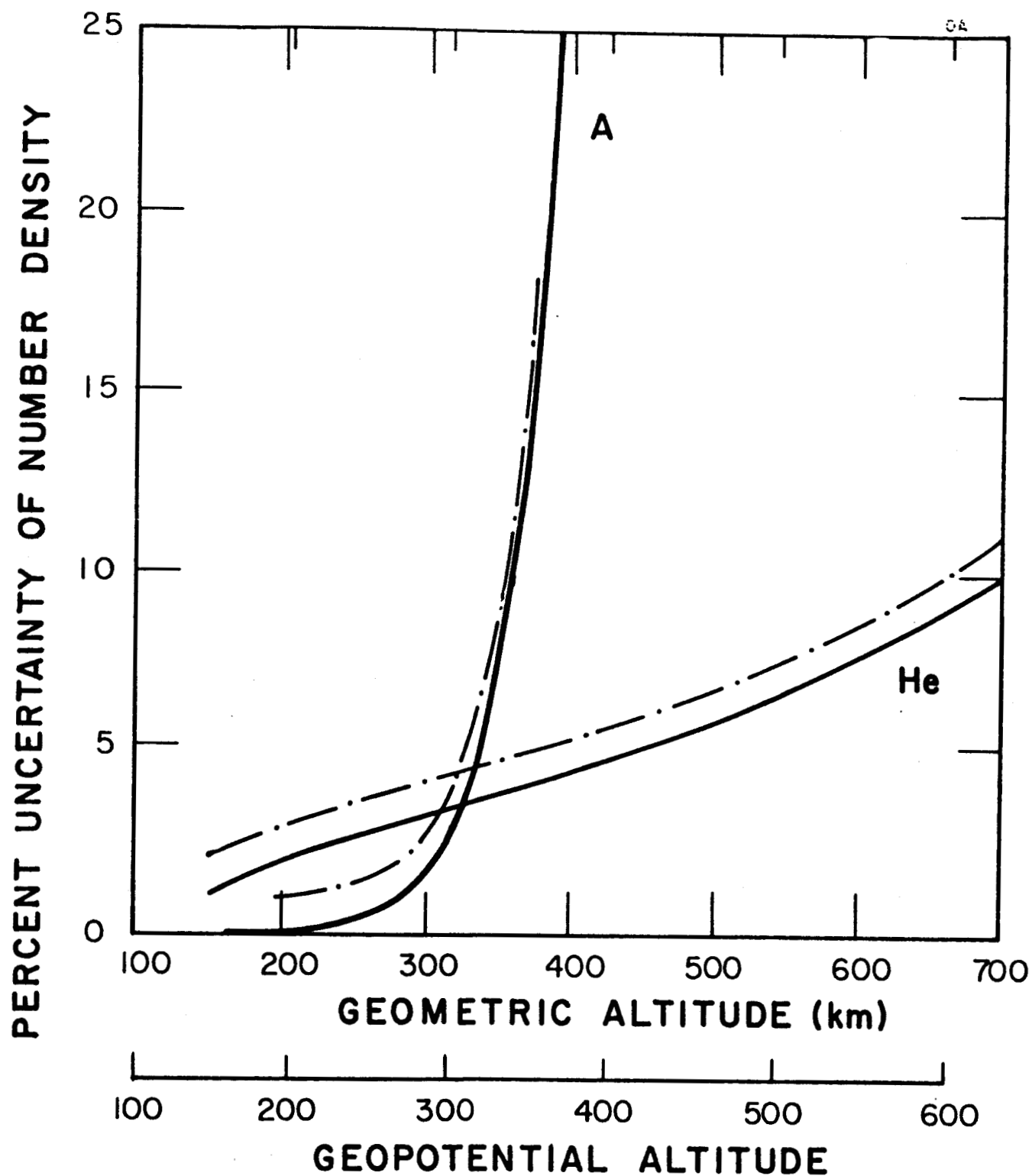


Figure 6. Percent uncertainty of helium and argon number densities vs altitude for various combinations of mass-spectrometer sensitivity and gas concentration, and a fixed reading error.

density profile and the assumed error function, this reversal point occurs near 400 km altitude, i.e., a reference altitude of 400 km yielded the lowest uncertainty curves for downward-type calculations.

The use of the limits 150 and 400 km in each of four equations yields a set of temperature uncertainties as shown in Figure 7. Here the uncertainty of T from the double-gas-up equation is represented by the heavy solid line which shows δT dropping from infinity at the reference altitude $b = 150$ km to about 60°C at 250 km and rising as the signal-to-noise ratio decreases with increasing altitude. The uncertainty from the double-gas-up equation at 400 km when introduced into the heavy-gas-down equations yields results shown in the dash-dot line, with errors dropping to about 10°C at 150 km. This value of δT introduced into the light-gas-up equation yields values of δT which are represented by the light dashed line and are similar to those of the double-gas-up expression above 250 km. The double-gas-down expression represented by the heavy solid line has uncertainties which start at infinity at the reference altitude and drop to values similar to those of the heavy-gas-down expression at low altitudes. It is apparent that the uncertainties in temperature for the double-gas-up equation and for the light-gas-up equation converge at altitudes above 250 km as do the forms of the related temperature equations. Similarly, the temperature uncertainties for the double-gas-down and heavy-gas-down equations converge at altitudes below about 250 km as do the forms of the related temperature expressions. The temperature uncertainties

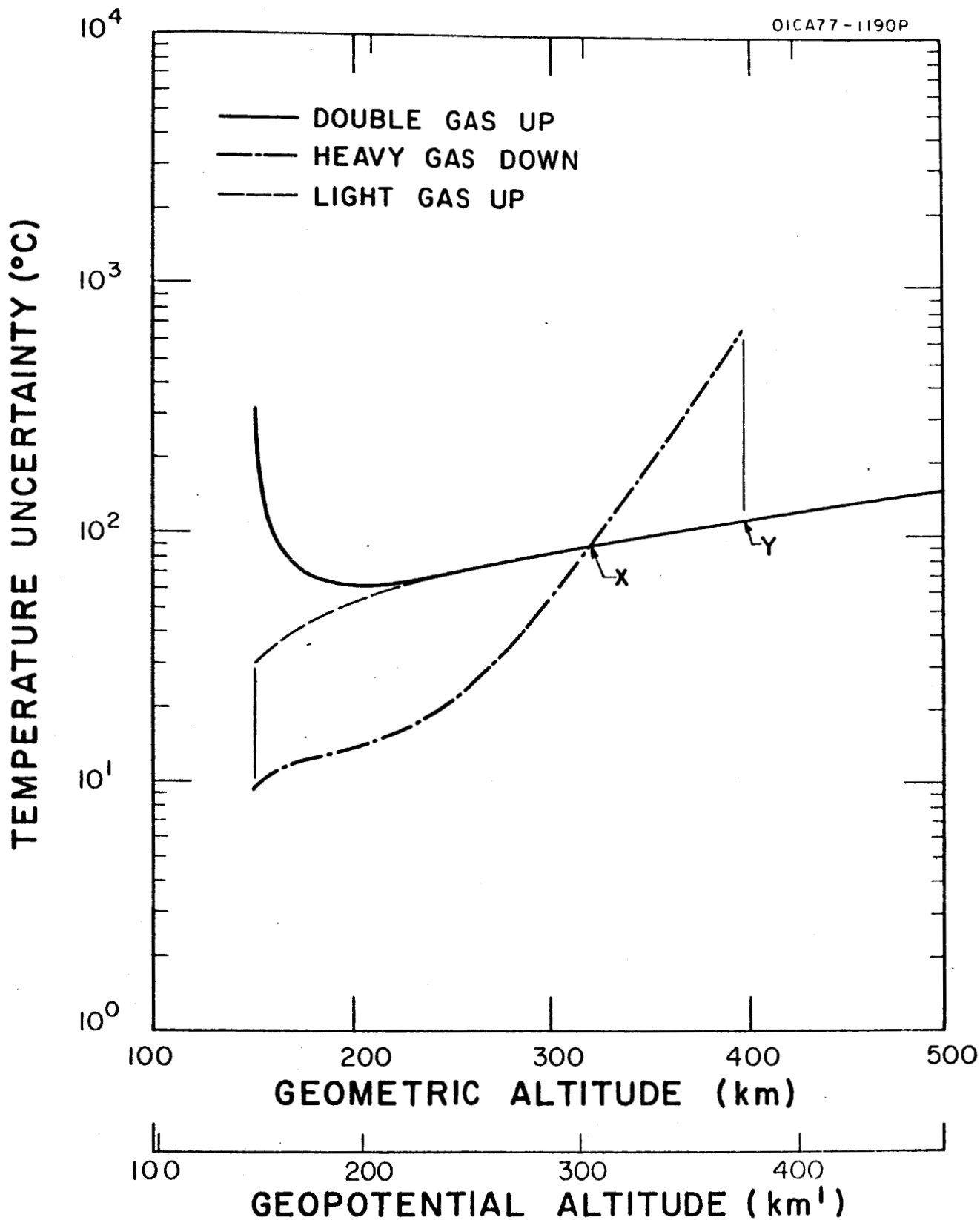


Figure 7. A comparison of the optimum uncertainty values for the double-gas-up equation, heavy-gas-down equation and the light-gas-up equation.

of all four equations are nearly equal at 325 km, the altitude at which the helium and argon number densities were assumed to be equal (See Figure 1). This situation is due in part to the assumption of equal mass-spectrometer sensitivities for both gases. At low altitudes the heavy-gas equations provide the smallest temperature uncertainties because of the greater heavy-gas number-density at these altitudes. At high altitudes the light-gas equations provide the smallest temperature uncertainty for similar reasons.

VI. APPROXIMATION ERROR EXPRESSIONS

By means of a numerical analysis of the various terms of the error equation it may be shown that certain simplification may be made. If the number-density observations are made at sufficiently small intervals such that $\Delta h \simeq 0.1$ km, those terms in u^2 of Equation (9) and in v^2 of Equation (10) which contain the factor Δh_j or Δh_a become negligibly small for any altitude region, and the expressions u^2 and v^2 may be simplified to

$$u^2 = (\delta n_a^* T_a)^2 + (\delta n_b^* T_b)^2, \quad (9a)$$

and

$$v^2 = (\delta n_a T_a)^2 + (\delta n_b T_b)^2. \quad (10a)$$

This simplification has a trivial effect on the value of δT from the light-gas-dominated equations, even if Δh is not made small, but does have a more significant effect on the value of δT associated with the heavy-gas-dominated equations. Consequently, in order to avail oneself of this simplification in data analysis the heavy-gas observations should be made at reasonably small altitude intervals (perhaps 1 km would be suitable). A numerical examination of the effects of various-sized intervals shows that with an interval of about 0.1 km Equations (9a) and (10a) yield values of δT which are essentially exact, and that for an interval of about 10 km the heavy-gas-dominated uncertainty equations are accurate to within about 6% of the exact value of δT .

It may also be shown that for altitudes sufficiently below the reference altitude "a", in the case of the heavy-gas, or heavy-gas-dominated expressions, Equations (5) and (8) may be replaced by the following very simple approximation:

$$\left(\delta T_b^a\right)_h \simeq \left(\delta T_b^a\right)_d \simeq \frac{\delta n_b^*}{n_b^*} T_b. \quad (13)$$

Similarly for altitudes sufficiently above the reference altitude "b" in the case of the light-gas, or light-gas-dominated expressions, Equations (6) and (7) may be replaced by the following very simple approximation:

$$\left(\delta T_{\underline{b}}^a\right)_\ell \approx \left(\delta T_{\underline{b}}^a\right)_d \approx \sqrt{2} \frac{\delta n_a}{n_a} T_a. \quad (14)$$

Thus, to a first approximation, the percentage uncertainty in the atmospheric temperature at any specific altitude (as deduced from the altitude-dependent number-density data of a light and heavy gas) is proportional to the percentage uncertainty of the appropriate number-density data at that altitude. The reliability of the approximations expressed by Equations (13) and (14) are shown in Figure 8. Separate lines represent the percentage uncertainty in temperature as computed from each of Equations (5), (6), (7) and (8) (where u^2 and v^2 are replaced by their exact expression). Additional lines represent the percentage uncertainty of argon and helium number-density data. The difference between the values of uncertainty at the same altitude indicate the degree of reliability of Equations (13) and (14). Figure 8 shows that these equations apply as very good approximations over limited altitude regions and as first order approximations over most of the altitude region considered. The approximations are least valid for the double-gas equations near their respective reference altitudes.

A finer measure of the reliability of the approximation equations is given in Figure 9 where the ratio of the approximation value of δT to the exact value of δT for each of the appropriate Equations (5), (6), (7) and (8) is plotted as a function of altitude. Two computations are

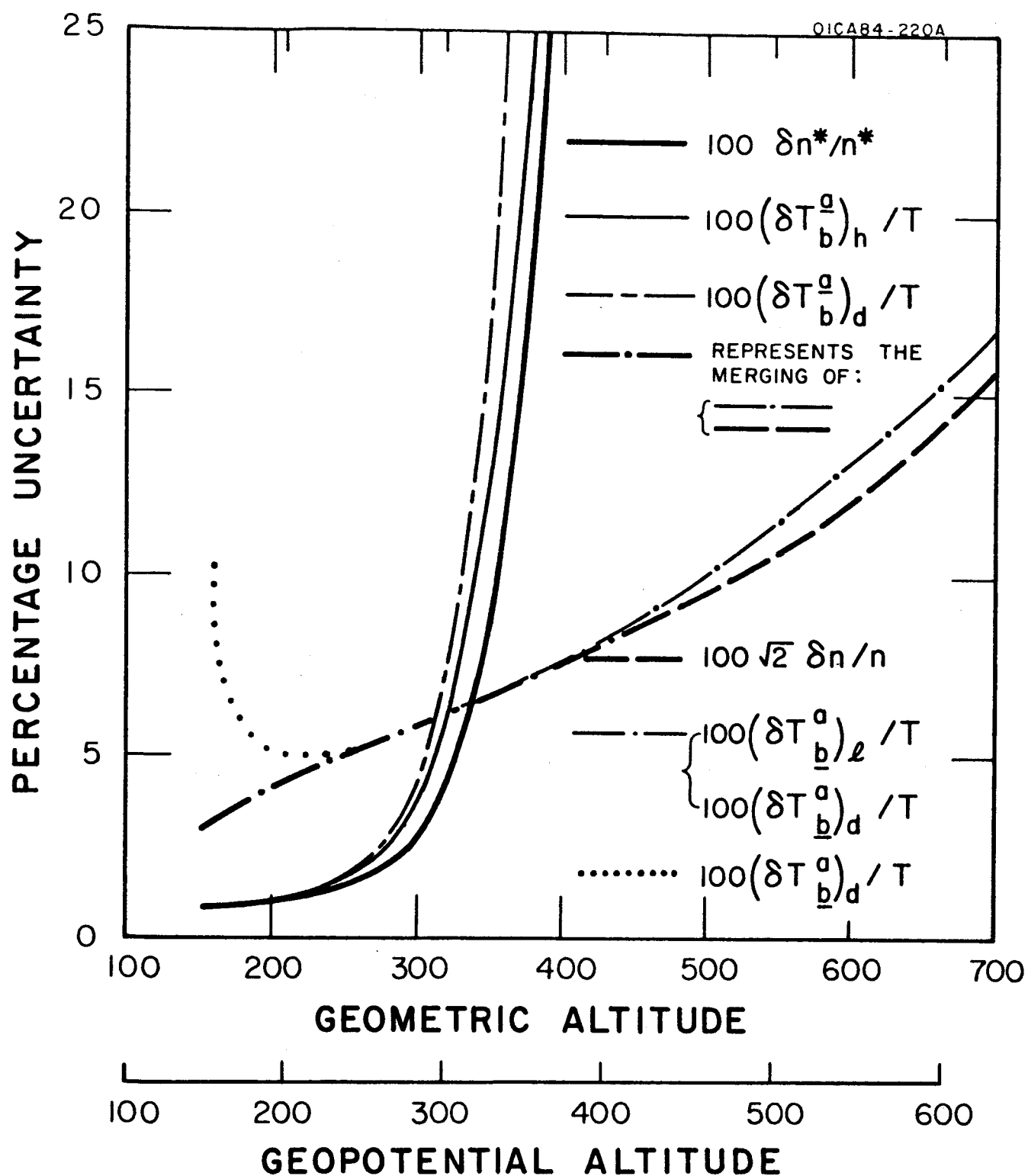


Figure 8. Percent temperature uncertainties as determined by the simplest approximation equations compared with percent number-density uncertainty of both helium and argon.

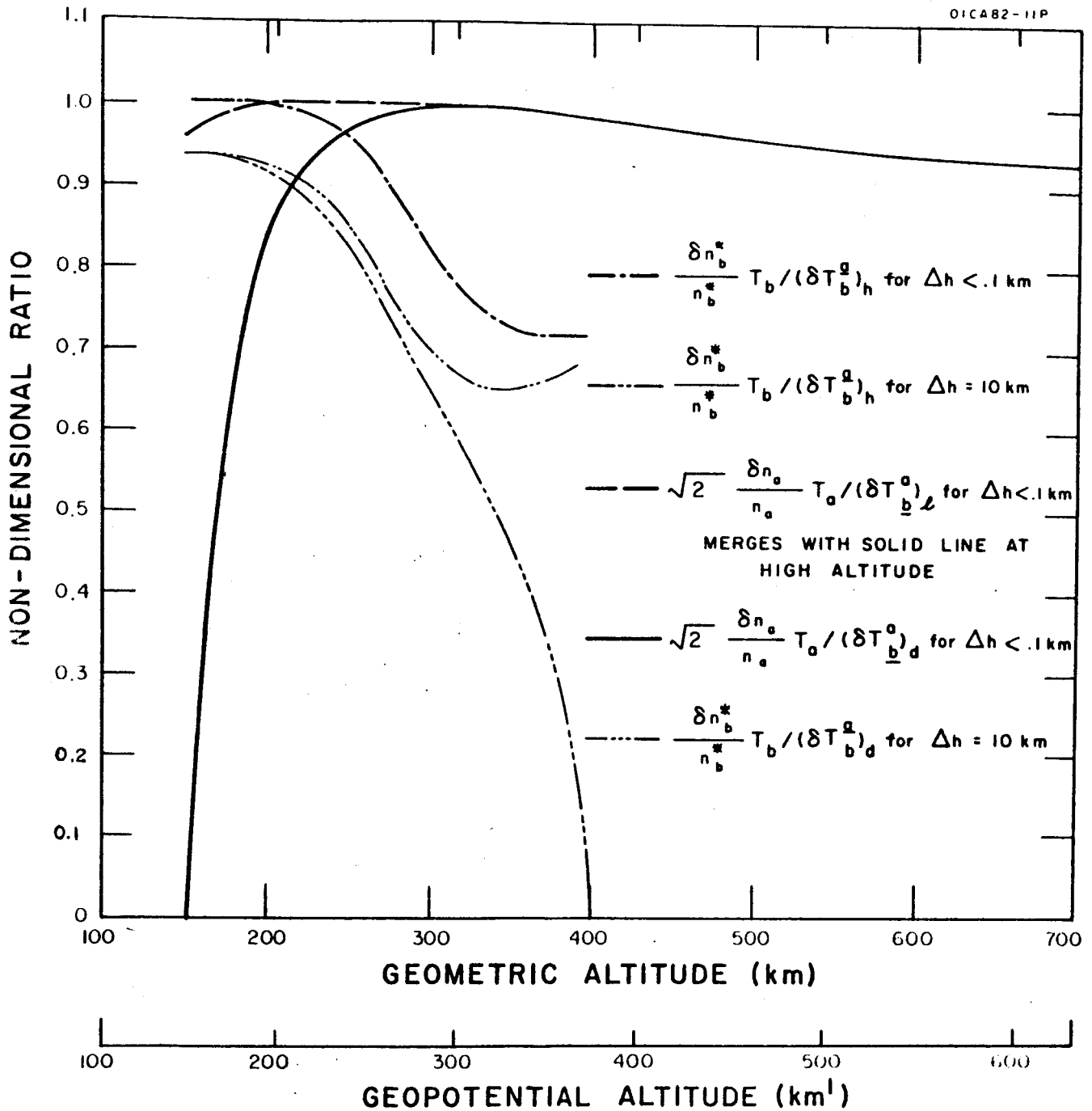


Figure 9. Ratio of temperature uncertainties as computed by simplest approximation equation to those computed by corresponding exact error equation.

presented for the ratio involving the heavy-gas-down equation; i.e., $\frac{\delta n_b^*}{n_b^*} T_b / (\delta T_b^a)_h$. One of these is for $\Delta h = 10$ km in Equation (9) and one for $\Delta h < .1$ km. Obviously, the simplest approximation expression Equation (13) most nearly expresses the correct uncertainty when the altitude interval between heavy-gas-number-density data is smallest.

VII. SUMMARY OF RESULTS

Figure 10 shows a graph of the temperature-altitude profile of the U.S. Standard Atmosphere 1962, with the loci of the minimum computed temperature uncertainties superimposed. At low altitudes where the mass-spectrometer output signal for argon number density n^* is more than two orders of magnitude greater than the assumed spectrographic sensitivity, it is the 1% reading error which determines the temperature uncertainty. At somewhat higher altitudes as the value of argon number density increases to the point where the inverse of the signal-to-noise ratio governs the fractional uncertainty in n^* , it is this inverse signal-to-noise ratio which determined the uncertainty in T . At still greater altitudes where the helium-dominated double-gas-up equation is used, the temperature uncertainty is governed approximately by the inverse of the signal-to-noise ratio of the mass spectrometer for helium. If the helium number density is found to be ten times greater than that speculated in Figure 1, the inverse signal-to-noise ratio will decrease by a factor of ten and the temperature uncertainty will decrease by a somewhat smaller amount, about a factor of 7. Increasing the sensitivity of the mass spectrometer would also serve to decrease the temperature uncertainty.

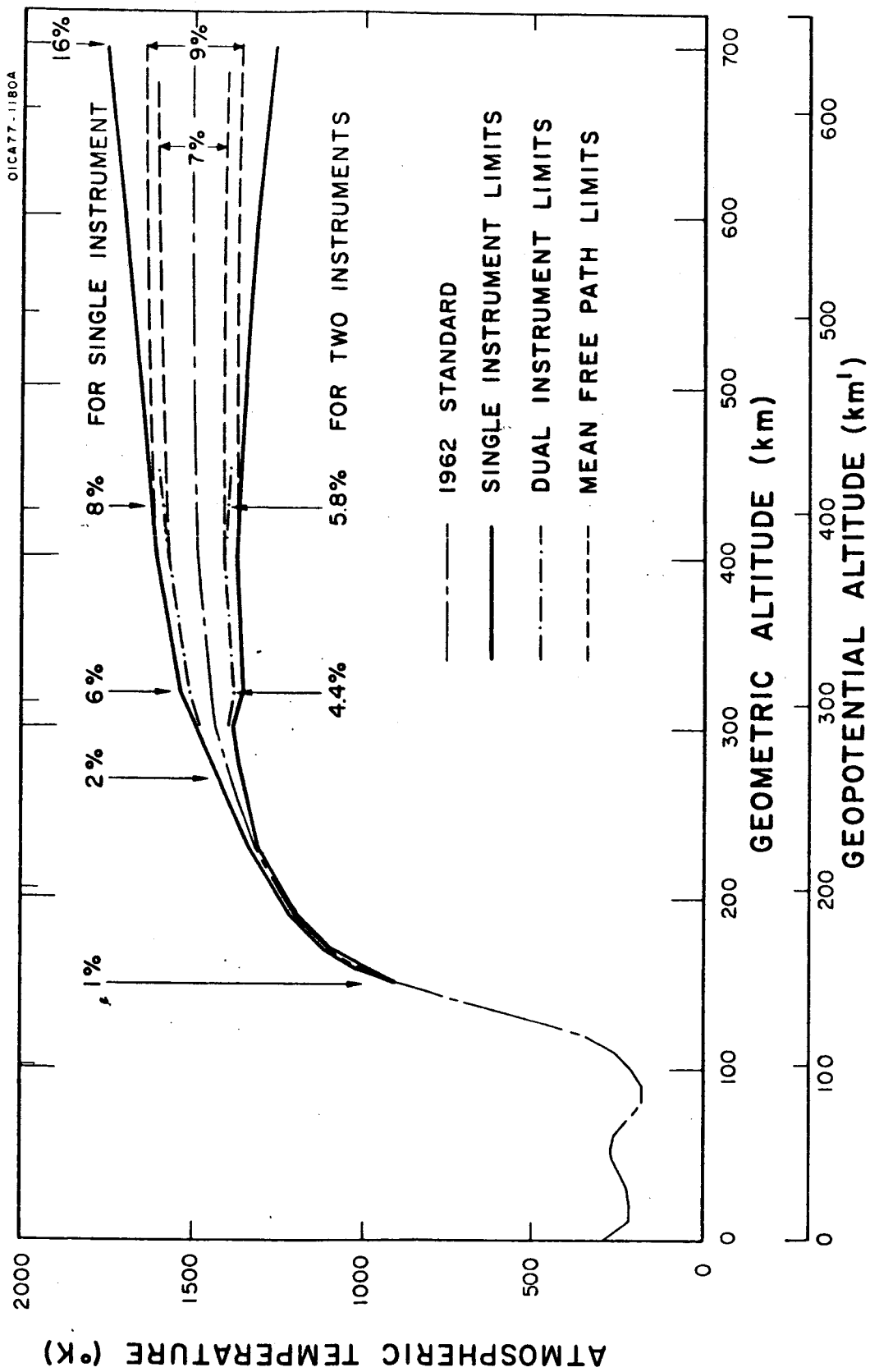


Figure Temperature altitude profile of U.S. Standard Atmosphere with superimposed temperature uncertainties as obtained by various equations.

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C. SUMMARY OF TECHNICAL REPORTS PREPARED ENTIRELY UNDER RELATED
AIR FORCE CONTRACT

NOTE ON EQUATIONS FOR COMPUTING NUMBER-
DENSITY RATIOS FROM TEMPERATURE PROFILES

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ABSTRACT

Equations giving the number-density ratios in an atmosphere characterized by non-zero linear temperature gradients and by an inverse-square law gravity field have been discussed in the literature.⁽¹⁾⁽²⁾ It is shown that these equations are, however, somewhat inconvenient to use in manual computations and cannot be handled properly by an electronic computer over certain regions of the atmosphere. A series solution which bypasses these difficulties and which can be applied accurately over the entire altitude range of a diffusively separated atmosphere has been derived. An iterative process in which the temperature profile is assumed linear in 5-km segments is described in terms of this series solution. One is enabled therefore to compute straightforwardly, number density ratios of some individual constituents in diffusive equilibrium in the upper atmosphere.

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SUMMARY

From a combination of the hydrostatic equation, the perfect gas law, and kinetic theory, one can obtain the well-known result

$$\ln \left(\frac{n_a T_a}{n_b T_b} \right) = - \int_b^a \frac{M(z) g(z)}{R T(z)} dz \quad (1)$$

Thus, given a temperature profile $T(z)$ and given $M(z)$ and $g(z)$ in some altitude range, the number density at any point a can be calculated in terms of a reference level b . For applications over small altitude increments it is sufficient to assume that $T(z)$ is a linear function of altitude. If in addition we assume diffusive separation and an inverse-square law for $g(z)$, i.e., if

$$T(z) = T_b + L(z - z_b)$$

$$M(z) = M_0$$

$$g(z) = g_0 \left(\frac{r}{r+z} \right)^2 ,$$

where

g_0 is the sea-level value of g

r is the effective mean radius of the Earth ,

integral (1) can be evaluated analytically. (1, 2) The resulting expression is

$$n = n_b \frac{T_b}{T} \left[\frac{T}{T_b} \frac{r+z_b}{r+z} \right]^{-\frac{\alpha L}{[T_b - L(r+z_b)]^2}} \quad (2)$$

$$\times \exp \left[-\frac{\alpha(z-z_b)}{T_b - L(r+z_b)} \frac{1}{(r+z_b)(r+z)} \right]$$

in which

n is the number density at altitude z above sea level

n_b is the number density at the reference altitude z_b

T is the kinetic temperature at altitude z

T_b is the kinetic temperature at the reference altitude z_b

α is the constant $\frac{M_o g_o r^2}{R}$

A real atmosphere has a temperature profile which is a well-behaved, smooth function of altitude. It is therefore reasonable to consider the profile as a series of small linear segments thereby allowing application of Equation (2) over each segment. Knowing the density of a constituent at some reference level, then, one can compute the density at any other level in a step-by-step process, provided diffusive separation is maintained throughout the interval. An iterative computation of this kind is well adapted to an electronic computer since the segments can be made small enough to permit a very good representation of the temperature profile.

Direct use of (2) in a computer is satisfactory only over certain ranges of ranges of values of the parameters T_b and L . The reason such a calculation may be unsatisfactory for other values, comes from the apparent blowup of the exponents when $T_b = L(r+z_b)$. Even when T_b is significantly different from $L(r+z_b)$, one is often in trouble because of the inherent limitations of the machine, in which significant digits are lost for some operations.

In order to bypass these difficulties, Equation (2) may be expanded in a converging series such that the points for which T_b approaches $L(r+z_b)$ do not cause trouble. The series solution derived from Equation (2) is

$$n = n_b \frac{T_b}{T} \exp \left[- \frac{\alpha(z-z_b)}{T_b(r+z)(r+z_b)} + \sum_{j=2}^{\infty} \frac{\alpha L(z-z_b)^j}{j(T_b)^j (r+z)^j} [T_b - L(r+z_b)]^{j-2} \right] \quad (3)$$

Equation (3) is always valid when altitude increments are made sufficiently small. For use in the earth's atmosphere, limiting $z-z_b$ to 5 km assures a rapid enough convergence of the series so that a computer can handle the calculation accurately for altitudes above 120 km. For altitudes below 150 km, about 10 terms are needed to get answers correct to eight significant digits for each segment; above 200 km, only about four terms are needed. Some loss of accuracy may occur in the region below 200 km because of a further limitation of

the computer, but this difficulty may be overcome by taking segments smaller than five kilometers. An appendix to the report describes a computer program set up for the series solution given as Equation (3).

A POLYNOMIAL FIT TO THE DEFINING
POINTS FOR MEAN MOLECULAR WEIGHT
IN THE 1962 U.S. STANDARD ATMOSPHERE

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ABSTRACT

Polynomials have been fitted to the 14 data points for mean molecular weight, $M(z)$, computed by Champion as defining points for the "U.S. Standard Atmosphere 1962" ⁽¹⁾ ⁽²⁾ One set of polynomials of relatively high order has been presented and then used to construct a table of values for $M(z)$. The results represent in some respects a better fit to the defining points than the fit used in the 1962 standard atmosphere. In addition, several polynomials, (orders one, two, and three) have been given and are particularly useful for general computations. These low order polynomials agree with the defining points to within about 1%.

The mean molecular weight as defined in the 1962 standard atmosphere is questioned, however, in the light of new data involving atmospheric composition determinations. The new data indicate lower values of $M(z)$ than are given by the standard atmosphere at intermediate altitudes.

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SUMMARY

Champion⁽¹⁾ has calculated the mean molecular weight at several altitudes in the interval 90-700 km. The values, reproduced in Table 1, were used as defining points in constructing the table of mean molecular weight in the "U.S. Standard Atmosphere, 1962".⁽²⁾ The tables for the 1962 standard were constructed by fitting polynomials through the defining points, but these polynomials were not explicitly given, and hence are not available for computations of parameters related to the standard atmosphere.

TABLE 1

DEFINING VALUES OF MEAN MOLECULAR WEIGHT OF THE 1962 STANDARD ATMOSPHERE

Altitude <i>z</i> (km)	Mean Molecular Weight <i>M</i>
90	28.9644
100	28.88
110	28.56
120	28.07
150	26.92
160	26.66
170	26.40
190	25.85
230	24.70
300	22.66
400	19.94
500	17.94
600	16.84
700	16.17

In connection with such calculations we similarly have fitted polynomials to the defining molecular-weight points and felt it useful to make these results available. One set of polynomials was made to conform as closely as possible, within reason, to the defining points. Recognizing that this is a rather artificial requirement except for purposes of a standard, a second set of polynomials of low order, which can be used for most calculations, has been included.

The high-order set of polynomials was computed by a modified least-squares fit and are presented as Equations (1) and (2).

For the altitude interval 90-147.5 km

$$M(z) = \sum_{i=0}^5 a_i (z-90)^i \quad (1)$$

in which

$$a_0 = 28.9644$$

$$a_1 = 0$$

$$a_2 = -4.19539 \times 10^{-4}$$

$$a_3 = -5.77605 \times 10^{-5}$$

$$a_4 = 1.65350 \times 10^{-6}$$

$$a_5 = -1.22385 \times 10^{-8}$$

For the altitude interval 147.5 - 700 km

$$M(z) = \sum_{i=0}^9 a_i z^i \quad (2)$$

in which

$$\begin{aligned}a_0 &= 29.996600 \\a_1 &= 1.5373738 \times 10^{-2} \\a_2 &= -3.4108510 \times 10^{-5} \\a_3 &= -1.5423610 \times 10^{-8} \\a_4 &= 8.9069500 \times 10^{-11} \\a_5 &= -5.9998820 \times 10^{-14} \\a_6 &= 3.7037890 \times 10^{-16} \\a_7 &= -3.0071000 \times 10^{-20} \\a_8 &= -1.3660042 \times 10^{-21} \\a_9 &= 1.1222209 \times 10^{-24} .\end{aligned}$$

Of the 14 data points given in Table 1, 11 are reproduced exactly to 4 significant digits, 2 are given to within 1 digit in the fourth place, and 1 is reproduced to within 2 digits in the fourth place. In addition the slope dM/dz is zero at 90 km altitude at which point $M=28.9644$. This fit to the defining points for mean molecular weight is similar to but better, in some respects, than the table given in the "U.S. Standard Atmosphere 1962".

Three sets of low-order polynomials which in general agree to within 1% of the standard atmosphere value are given in Table 2.

TABLE 2

APPROXIMATE POLYNOMIALS FOR MEAN MOLECULAR WEIGHT
AS DEFINED BY THE 1962 U. S. STANDARD ATMOSPHERE

Altitude Range (km)	Polynomial for Mean Molecular Weight $M(z)$	Maximum Deviation From Standard
0-90	28.9644	0%
90-350	Set 1 { 31.666 - .03002 z 28.905 - .02213 z 22.19 - .0087 z	$\leq .8\%$
350-500		$\leq .8\%$
500-700		$\leq .8\%$
0-97	Set 2 { 28.9644 32.807 - .042338 z + 2.6264 x 10 ⁻⁵ z ²	$< .2\%$
97-700		$\leq 1.5\%$
0-93	Set 3 { 28.9644 32.154 - .034513 z + 2.0326 x 10 ⁻⁶ z ² + 2.1032 x 10 ⁻⁸ z ³	$<< .1\%$
93-700		$\leq .7\%$

Recent atmosphere composition determinations from mass spectrometer measurements (Schaefer, 1963)⁽³⁾ and from EUV and X-ray absorption measurements (Hinteregger, 1962)⁽⁴⁾ and (Kupperian et al., 1960)⁽⁵⁾ suggest that the O_2 dissociation is considerably different from that used by Champion in his preparation of the standard-atmosphere definition of $M(z)$. Consequently, the added refinements which our polynomials provide are only of academic interest scientifically. These polynomials do, however, provided others with the means for making calculations of the quasi-legal type associated with the standard atmosphere, as well as the means for obtaining improved results.

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III. CONCLUSIONS AND RECOMMENDATIONS

The first Scientific Report under the contract, giving a history of standard atmospheres, presents a picture of how we got to where we are in atmospheric models with ever increasing complexity. The need for perpetual vigilance is indicated to avoid difficulties which will accelerate the obsolescence of the current or future standard atmospheres.

One way of minimizing difficulties and obsolescence in standard atmospheres is to properly establish a set of basic constants, for defining a standard atmospheres. This problem was discussed in part in the Second Scientific Report published initially as a Technical Note to the members of COESA, and now republished with some modifications. Equations were presented for the calculation of atmosphere properties in terms of the following three sets of basic defined constants and functions

1. $p_o, T_M(h), M_o, R^*$;
2. $p_o, T_M(h), \rho_o, R^*$, and
3. $p_o, T(h), M(h) R^*$.

While standard atmospheres in recent years have been defined in terms of molecular scale temperature, for want of adequate knowledge concerning kinetic temperature alone, the progress in the field of measurement may change this situation and $T(h)$ may be once again the basic parameter, even at high altitudes.

This possibility is suggested as a result of the work presented in Scientific Report 4 on this contract which presents a kind of breakthrough in atmospheric-temperature determination. This report presents the details of a method for the accurate determination of temperature over the entire altitude region where helium and argon number-density data may be accurately measured, and at any altitudes above this region where only helium data are obtainable. This method is revolutionary since unlike any other method suggested for practical use at great altitudes, in the region of diffusive separation, it alone determines T rather than T/M .

The detailed error analysis is also rather unique since little or no such approach is seen in the literature, and one must infer that frequently little rigorous consideration is given to many of the planned atmospheric measuring experiments. As a result of the error analysis performed, one not only sees the need for adequate sensitivity of the sensing element, but, for particular sensitivities, one can predict with precision the uncertainties of the temperatures computed by any of the various equations involved.

The utilization of the method described should eliminate many of the uncertainties in atmospheric models on the regions above 120 km. By means of rocket probes using the method described, one can obtain essentially instantaneous altitude profiles of $T(h)$, the altitude of the micropause, and, hence, the means for a much more precise determination of composition, for various times of day, season or location.

Scientific Report #3 is a very condensed summary of recent work in the field of direct probing experiments and serves as a basis for seeing the state of the art as well as for obtaining data for an analysis program which is underway but not completed to the reporting point. It is hoped that this work will be completed in a continuation of the contract being reported herein.

density-data points. The earth, of course, rotates about 22° per 96 minutes orbital period so that each orbital pass covers a very slightly different region of longitude. There is essentially no change in the related altitude for successive satellite passes through perigee except insofar as the earth may not be spherical.

The resolution of altitude and coordinate data associated with computed density-data points is poor, and the time rate of change of altitude or latitude is small. Thus, only over long periods of time ranging from weeks to months or even years are there significant changes in the mean values of latitude and altitude associated with the data. The drag data of a single satellite will therefore yield only isolated data points from which it is impossible to obtain anything approaching an instantaneous density-altitude profile at a fixed latitude or an instantaneous density-latitude profile at a fixed altitude.

Successive perigee passes do produce data with longitude increments of about 22° but having but very little change with respect to the local sun time of the various sub-perigee points.

The satellite-drag method for measuring density is best suited to the determination of time dependent variations of density within a diffuse region characterized by essentially constant altitude, constant latitude and constant local sun time. Thus, the method is rather well suited to observing density changes at one point related to solar radiation phenomena. The method does not provide spatial distribution information, however.

The simultaneous observation of the drag of n satellites each with a different perigee position and different perigee-passage time could at best provide density data for n different space regions over a 90-minute period and these space regions would have a poor resolution. The generation of latitude-dependent, and altitude-dependent, fine-structure models of the atmosphere from drag-related density data alone is a questionable procedure. Results from instrumented rocket and satellite probes must be included for adequate interpretation.

Even within the drag method there are some questions, which may be raised with respect to the appropriate drag coefficient to be used in computing mass density from drag acceleration for bodies of various non-spherical shapes.

D. SCOPE OF PROPOSED PROGRAM OF INVESTIGATION

Several of the above questions suggest very large areas of investigation entirely outside of the size conceived by this proposal. The scope of work suggested by this proposal includes the detailed study of some of the lesser of the above questions plus the surveying of the work of others in these more extensive areas such that all the results may be applied to the interpretation of aeronomy data and to the development of time-and-space-dependent models of the earth's atmosphere. It is also proposed that several areas included in the current investigation be continued.

Some of the questions examined separately and jointly on the NASA contract and on the related Air Force contract during the past year include the following:

- (1) History of standard and model atmospheres
- (2) Self-consistent sets of constants for standard and model atmospheres
- (3) Development of an analytical function of molecular weight versus geometric altitude.
- (4) Development of a new form of the number-density-versus-altitude equation which does not become indeterminate for critical values of constant temperature gradients.
- (5) Survey of recent results of atmospheric probing experiments
- (6) Cataloging of atmospheric density data in the 90 to 200 km region for use in preparing supplemental model atmospheres
- (7) Determination of inherent accuracy of several methods for high-altitude temperature determination.

Some of the above areas of interest and investigation are completely finished units of work as indicated by the technical reports, while other phases are still in rough form.

In particular, these uncompleted or continuing phases include the development of supplementary model atmospheres and the error study of various methods to temperature determination both of which would be continued along with specialized studies as directed by the contract monitor.

IV. STATEMENT OF WORK

The Contractor shall furnish all necessary personnel, facilities, materials and equipment and shall use its best efforts to perform, under the direction of Mr. Raymond Minzner as Principal Investigator and Staff Scientist, the tasks specified below:

Task A. Attend and actively participate in, as a NASA representative, all meetings of the Committee on Extension to the Standard Atmosphere and all COSPAR meetings pertaining to discussions of standard atmospheres. In addition, the Contractor shall conduct special studies and prepare special reports, as required by the Government, incidental to such attendance and participation.

Task B. Study and analyze all existing data relative to the preparation of a revised standard atmosphere and various supplementary atmospheres.

Task C. Gather, analyze and study atmospheric and other data pertaining to the development of new knowledge of the average structure and variability of the earth's atmosphere above the level routinely investigated by meteorological balloon techniques.